MA261 Quiz 4

July 5, 2016

Problem 1.

Find and classify the critical points of the function

$$f(x, y) = x^3 - 3xy^2 + 3y^2 + 9x$$

Solution.

$$f_x = 3x^2 - 3y^2 + 9 \tag{1}$$
$$f_y = -6xy + 6y = 6y(-x + 1) \tag{2}$$

By (2), either $y = 0$ or $x = 1$. When $y = 0$, $f_x = 3x^2 + 9 > 0$, so the critical points are $(1, 2)$ and $(1, -2)$. Now,

$$f_{xx} = 6x$$
$$f_{yy} = -6x + 6$$
$$f_{xy} = -6y$$

In both cases, $D = f_{xx}f_{yy} - (f_{xy})^2 = -144 < 0$, so they are all saddle points.

Problem 2.

Find the directional derivative of f at the point $(2, 6)$ in the direction of the vector $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j}$.

$$f(x, y) = \sqrt{x^2 + xy}$$

Solution.

$$f_x = \frac{2x + y}{2\sqrt{x^2 + xy}} = \frac{5}{4}$$
$$f_y = \frac{x}{2\sqrt{x^2 + xy}} = \frac{1}{4}$$

A unit vector in the direction of $\mathbf{u}$ is $\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$. So the directional derivative $D_{\mathbf{u}} f(2,6) = (\frac{5}{4})(\frac{1}{\sqrt{2}}) + (\frac{1}{4})(-\frac{1}{\sqrt{2}}) = \frac{1}{\sqrt{2}}$.